Effective Computational Geometry
Exercise 3

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Exercise 1: The *side of circle predicate* applies to four points (p, q, r, x) in the plane. It returns $+1$ if x is to the left of the oriented circle through p, q, and r, returns -1 if x is to the right of the oriented circle through p, q, and r, and returns 0 if either $|\{p, q, r\}| \leq 2$ or x lies on the oriented circle through p, q, and r. We give some more explanations. Three points p, q, and r that are not collinear define a unique circle passing through them. We give this circle an orientation by insisting that p, q, and r occur in this order on the circle. Consider now a fourth point x. It is either left of, on, or right of the oriented circle through p, q, and r.

1. (5 pts) Give some examples to illustrate the definitions.
2. (5 pts) How should one define *side_of_circle*, if the first three points are collinear (but pairwise distinct).
3. (5 pts) Take $p = (-1, 0)$, $q = (1, 0)$, and $r = (0, 1)$. What is C? Verify the following formula. Hint: Evaluate the determinant

$$\mathit{side\_of\_circle}(p, q, r, s) = -\mathit{sign} \det \begin{vmatrix} 1 & 1 & 1 & 1 \\ p_x & q_x & r_x & s_x \\ p_y & q_y & r_y & s_y \\ p_x^2 + p_y^2 & q_x^2 + q_y^2 & r_x^2 + r_y^2 & s_x^2 + s_y^2 \end{vmatrix}$$

4. (5 pts) Interchange the role of q and r. What happens?
5. Take $p = (-1, 0)$, $q = (0, 0)$, and $r = (1, 0)$. What is C? Does the result of the formula agree with your answer to part (2). If not, do you want to reconsider your answer to part (2).
6. (10 pts) Prove the formula for general p, q, and r.

Exercise 2 Take the determinant formula from the preceding exercise. Develop a floating point filter for it. Hint: follow the example on page 36 of your handout.

1. (10 pts) Assume that the points are given by their Cartesian coordinates and that the Cartesian coordinates are integers.
2. (10 pts) Extend to integral homogeneous coordinates.

Solutions are due November 22nd and will be collected in the lecture on November 22nd. Solutions will be discussed in the tutorials of the week after.