Effective Computational Geometry
Exercise Sheet 5

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Exercise 1: Read the handout on Newton's method.

The handout will be discussed in the tutorials next week. The solutions for the other exercises are due December 20th and will be collected in the lecture on December 20th. Solutions will be discussed in the tutorials in the first week of January.

We have remodeled rooms 410 and 411. Room 410 is now a working and discussion room for students, room 411 is a terminal room.

Exercise 2: Show how to compute a^n with $O(\log n)$ multiplications.

Exercise 2: Compute Euler's number with maximal error 2^{-m} using the definition

$$e = \lim_{n \to \infty} (1 + \frac{1}{n})^n \ .$$

Hint: Exploit the fact that $(1 + 1/n)^n \leq e \leq (1 + 1/n)^{n+1}$ for all n.

Exercise 3: Implement your solution and time it for $m = 10000$. KM's program takes about 2.25 seconds on his notebook. Are you competitive? You might be able to answer this question without implementing your solution.

Exercise 3: In class, we discussed Vincent's algorithm for determining the number of real zeroes of a polynomial in the interval $(0, 1)$. Modify it to compute the number of positive real zeroes, the number of all negative real zeroes, and the number of all zeroes. There are two possible ways to attack these problems. I outline the attacks for the positive roots.

- Assume you want to determine the number of all non-nonnegative real roots of Q. Construct P such that the positive real roots of Q correspond to the real roots of P in $(0, 1)$ and then apply the algorithm presented in class

- Develop a variant of the algorithm which directly determines the number of positive real roots.

Follow the first approach for the positive and negative real roots. Follow the second approach for the positive roots. Hint: If $V(P) > 1$, split the interval $(0, \infty)$ into the subintervals $(0, 1)$ and $(1, \infty)$.

Exercise 4: An isolating interval for a real root α of polynomial P is an interval I with $\alpha \in I$ and I contains no other root of P.

- Show how to modify Vincent's algorithm so that it computes isolating intervals for all real roots of a polynomial.
- Apply the modification to the polynomial $x^3 - 7x + 7$.