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Channel Routing in Knock-Knee Mode: Simplified Algorithms and Proofs

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Abstract. We give unified and simplified algorithms and proofs for three results on channel routing in knock-knee mode. Let P be a channel routing problem with density $d_{\max}$.

(a) [Rivest/Baratz/Miller, Preparata/Lipski]. If all nets in P are two-terminal nets then $d_{\max}$ tracks suffice.
(b) [Preparata/Sarrafzadeh]. If all nets in P are two- or three-terminal nets then $\lfloor 3d_{\max}/2 \rfloor$ tracks suffice.
(c) [Sarrafzadeh/Preparata]. $2d_{\max}-1$ tracks always suffice.

In all three cases a solution can be found in linear time; this is an improvement in case (b).

Key Words. Layout technique, Channel routing problems, Knock-Knee layout mode, K-terminal nets.

Channel routing (and more general routing problems) have received considerable attention in the literature. Whilst earlier papers concentrated on heuristic algorithms there has been more emphasis on analysis in recent years [1–10]. In particular, it was shown that routing problems for two-terminal nets can be solved optimally and efficiently in many situations [1–5, 7]. For problems with multiterminal nets the situation is less clear. There are no optimal efficient algorithms known. However, there are approximation algorithms which come within a factor of $\frac{3}{2}$ of the optimum for three-terminal net problems [8] and within a factor of 2 of the optimum for all problems [9]. In this paper we give a unified and simplified treatment of these results. We show that a simple sweep algorithm can be used to obtain all results mentioned above and that a uniform method of analysis applies. Furthermore, the sweep algorithm always runs in time $\theta(n)$. This is an improvement in the case of three-terminal nets; the algorithm of [8] has running time $O(d_{\max}n)$ where $d_{\max}$ is the density of the problem.

A channel routing problem is specified by a set $\{N_1, \ldots, N_m\}$ of nets. A net N is a subset of $\mathbb{N} \times \{\text{bottom, top}\}$; it is called a k-terminal net if $|N| = k$ $(k \geq 2)$. The elements of a net are called its terminals or pins. We require that the nets in a channel routing problem are pairwise disjoint, i.e., $N_i \cap N_j = \varnothing$ for $i \neq j$. A solution in t tracks of a channel routing problem $N_1, \ldots, N_m$ consists of pairwise edge-disjoint connected subgraphs $T_1, \ldots, T_m$ of the integer grid

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$\{(x, y); -\infty < x < \infty, 1 \leq y \leq t\}$ such that $(i, 1)$ is a vertex of T_j if $(i, \text{bottom}) \in N_j$ and (i, t) is a vertex of T_j if $(i, \text{top}) \in N_j$. Figure 1 shows a channel routing problem and a solution in 4 tracks. In this figure the terminals of net N_i are labelled i.

The density of a channel routing problem is defined as follows. For $c \in \mathbb{N}$ let

$$d(c) := |\{N; N \text{ is a net with } N \cap ([1 \cdots c] \times \{\text{top, bottom}\}) \neq \emptyset \text{ and } N \cap ([c+1 \cdots n] \times \{\text{top, bottom}\}) \neq \emptyset\}|$$

be the number of nets which have to cross the vertical line $x = c + \frac{1}{2}$. Then

$$d_{\max} = \max\{d(c); c \in \mathbb{N}\}$$

is the density of the problem. It is clear that $d_{\max}$ is a lower bound on the number of tracks needed in any solution.

THEOREM. *Let P be a channel routing problem of density $d_{\max}$.*

(a) ([6, 7]). *If all nets are two-terminal nets then $d_{\max}$ tracks suffice.*
(b) ([8]). *If all nets are two- or three-terminal nets then $\lfloor 3d_{\max}/2 \rfloor$ tracks suffice.*
(c) ([9]). $2d_{\max} - 1$ *tracks always suffice.*

In all three cases a solution can be constructed in time $\theta(n)$.

PROOF. We use very similar algorithms for all three cases. In fact, the algorithm for case (b) can also be used for case (a). We nevertheless treat case (a) separately because it allows us to demonstrate our approach in a particularly simple case.

The algorithms process the channel in a single left-to-right sweep. The sweep stops in every column and performs a layout action. The layout action is usually defined by a case analysis. The algorithms have the following common characteristics.

C1. If the current column contains no pin or contains both pins of a two-terminal net then all wires coming into the current column from the left are extended by one unit. We may therefore assume without loss of generality that there are no empty columns and no trivial nets.

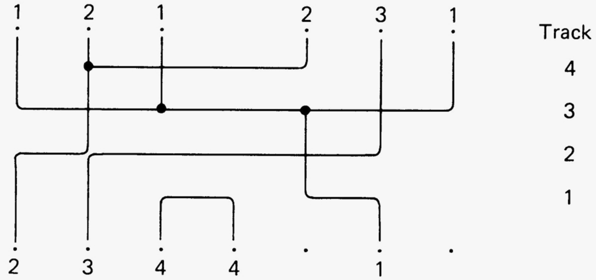


Fig. 1. A channel routing problem.

C2. A net occupies tracks only to the right of its leftmost pin. It may however use tracks to the right of its rightmost pin.

We employ the following terminology. If a net has terminated to the left of the current column, i.e., all its pins are to the left of the current column, and the net still occupies tracks then the net is called an *extended net.* An extended net always occupies at least two tracks. All other nets which occupy tracks are called active. For an active net the current column is between its leftmost and rightmost pin. A pin is called starting (terminating, continuing) if it is the leftmost (rightmost, neither of above) pin of a net. We are now ready for the algorithms.

(a) Two-Terminal Nets

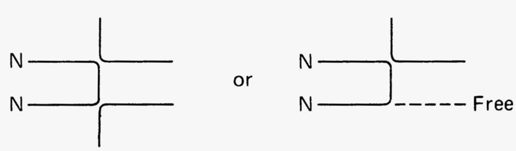
We will maintain the following invariant:

I1. Active nets use one track, extended nets use two tracks.
I2. $d+2e \leq d_{max}$, where d is the current density (=number of active nets) and e is the number of extended nets.

Note that the invariant directly implies that at most d_{max} tracks are used. It remains to describe the layout actions in the current column. We will verify the invariant in the comments of the algorithm. We use d to denote the density before the current column and d' to denote the density after the current column, similarly for e.

Case 1. Only starting pins in current column.

if there is an extended net N
then close the extended net and use its tracks for the starting nets



(*Comment*: $d' \leq d+2,\ e' = e-1$.)
else use a free track for every starting net
(*Comment*: $d' \leq d_{max},\ e' = 0$.)

Case 2. At least one terminating pin in current column.

Close a terminating net and use its track for the other net (if any) which has a terminal in the current column.

(*Comment*: let N' be other net having a pin in the current column. If N' does not exist then $d' = d - 1$, $e' = e$. If N' exists then either $d' = d$, $e' = e$ if N' is starting or $d' = d - 2$, $e' = e + 1$ if N' is terminating.)

The algorithm clearly spends time $O(1)$ per column and therefore runs in time $\theta(n)$.

(b) Two- or Three-Terminal Nets

We will slightly abuse language in this part. If a net N has two pins in the same column then we call one of them a continuing pin and the other one a starting or terminating pin (recall that we deal with three-terminal nets). We will maintain the following invariant.

I1. Active nets use at most two tracks. If an active net uses two tracks then exactly two of its terminals are to the left of the current column. Extended nets use at least two tracks.

I2. Active nets are paired or unpaired. In a pair exactly one net is double-stranded (uses two tracks) and the other one is single-stranded. Also both nets have exactly two terminals to the left of the current column and the single-stranded net is no shorter than, i.e., does not end before, the double-stranded net.

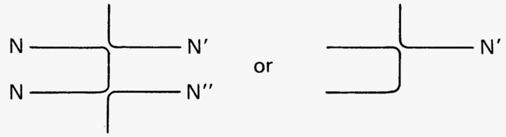
I3. $d + 2e + ud \le d_{\max}$, where d is the current density, e is the number of extended nets and ud is the number of unpaired double-stranded nets.

The algorithm to be described also has the property that it asks for a free track only if $e = 0$. From this and part 3 of the invariant it follows that at most $3d_{\max}/2$ tracks are used. Namely, let k be the number of pairs. Then $k \le d_{\max}/2$ since all paired nets are active. Also, the number of tracks used is $d + 2e + ud + k \le d_{\max} + k$, i.e., by invariant 3 it is at most $3d_{\max}/2$.

We are now ready to describe the layout actions in the current column. We leave it to the reader to check parts 1 and 2 of the invariant.

Case 1. No terminating pin in current column.

if there is extended net N
then close the extended net and use its tracks for the incoming nets N' (and N'')



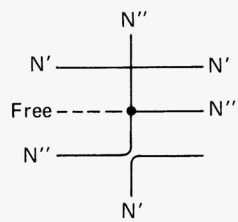
(*Comment*: in order to show that the invariant is maintained we distinguish cases:
N' and N'' are starting, $N' \ne N''$: $d' = d + 2$, $ud' = ud$;
N' and N'' are starting, $N' = N''$: $d' = d + 1$, $ud' = ud + 1$;

N' is starting, N'' does not exist: $d' = d + 1$, $ud' = ud$;
N' is starting, N'' is continuing: $d' = d + 1$, $ud' = ud + 1$;
N' is continuing, N'' does not exist: $d' = d$, $ud' = ud + 1$;
N' and N'' are continuing: $d' = d$, $ud' = ud + 2$;
in each case the invariant is maintained since $e' = e - 1$.)

else if all pins in current column are continuing
then if only one pin in current column
then make the connection

d'=d, e'=e, ud'=ud

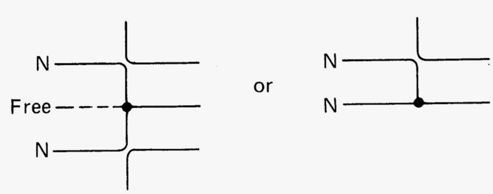
(*Comment*: $d' = d$, $e' = e$, $ud' = ud$.)
else use one additional track and make the shorter continuing net double-stranded



N' is not longer than N''

(*Comment*: we pair N' and N''; then $d' = d$, $e' = 0$, $ud' = ud$.)
fi

else (*Comment*: at least one starting pin in current column)
if there is an unpaired double-stranded net
then let N be an unpaired double-stranded net such that N is either the only unpaired double-stranded net or there is an unpaired double-stranded net $\bar{N}$ which is at least as long as N
if all pins in current column are starting
then use one additional track if there are two starting nets and no additional track otherwise; reduce N to a single track and make the starting net(s) single-stranded

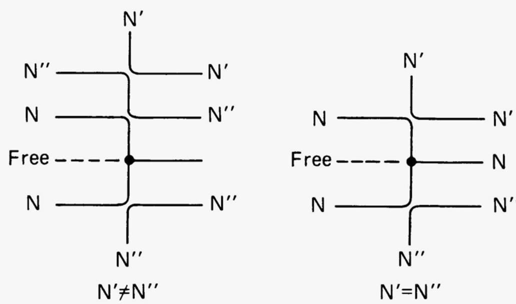


(*Comment*: if $\bar{N}$ exists then we pair N and $\bar{N}$. We have $d' \leq d + 2$ and either $ud' = 0$ (if $\bar{N}$ does not exist) or $ud' = ud - 2$. Also $e' = 0$ in both cases. Thus the invariant is maintained.)

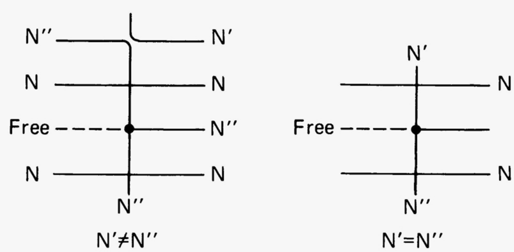
else (*Comment*: the pins in the current column belong to a starting net N' and a continuing net N''; $N' = N''$ is possible.)

if N'' is shorter than N

then use one additional track, make N single-stranded and N'' double-stranded



else use one additional track, leave N'' single-stranded and leave N untouched



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(*Comment*: in either case we pair N'' and N. Thus $d' = d + 1$, $e' = e$, $ud' = ud - 1$.)

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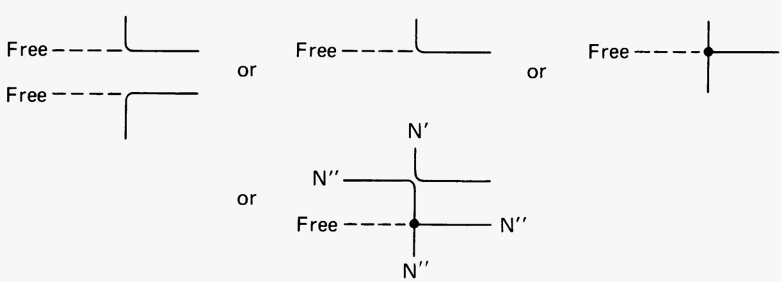
else (*Comment*: $ud = 0$)

use an additional track for every starting net)



(*Comment*: in either case $e' = ud' = 0$, $d' \le d_{\max}$.)

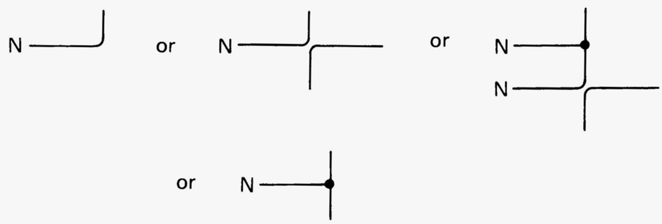
fi

This finishes the description of Case 1.

Case 2. At least one terminating pin in current column.

Let N be a net which ends in the current column. If two nets end in the current column and one of them is double-stranded then let N be double-stranded.

Terminate net N and use one of its tracks for the other net (if any) having a terminal in the current column.



(*Comment*: let N' be the other net (if any) which has a terminal in the current column and let N'' be the other member of the pair containing N (if N is paired); $N' = N''$ is possible. We observe first that N'' is single-stranded if N'' exists. This follows from the fact that the single-stranded member of a pair is no shorter than the double-stranded member and that N is chosen double-stranded if possible. Hence N'' does not become a double-stranded unpaired net. We can now show by a simple case analysis that the invariant is maintained.

N' does not exist: $d' = d - 1$, $e' = e$, $ud' = ud$;

N' is starting: $d' = d$, $e' = e$, $ud' = ud$;

N' is terminating: $d' = d - 2$, $e' = e + 1$, $ud' = ud$;

N' is continuing: $d' = d - 1$, $e' = e$, $ud' = ud + 1$;

$N' = N$: $d' = d - 1$, $e' = e$, $ud' = ud$).

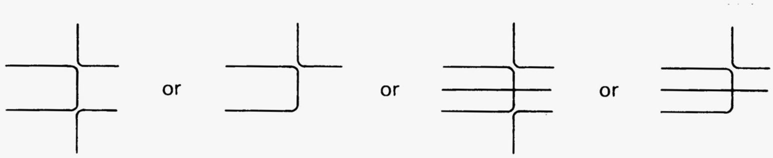
This finishes the description of Case 2.

The layout action in the current column clearly has cost $O(1)$; hence running time is $\theta(n)$.

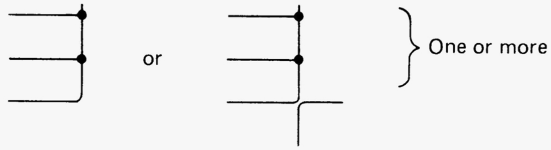
(c) Multiterminal Nets

Part (c) is the simplest to prove. The action in the current column is as follows.

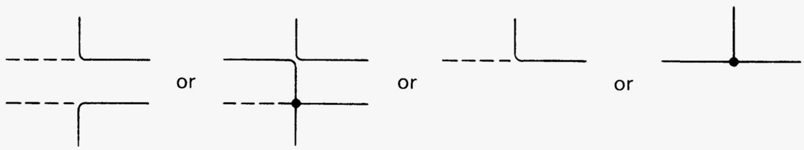
if there is an extended net or an active net which uses three or more tracks
then close the extended net or reduce the active net to a single strand and use the tracks for the incoming nets



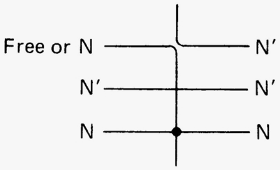
else if there is a terminating pin
then terminate a net and use its track for the other incoming net (if any)



else if at least one pin is starting
then use a free track for every starting net



else —both pins are continuing
if the two continuing nets have together only two tracks
then use an additional track
fi;
route using three tracks for the two nets



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(*Comments*: active nets use at most two tracks each. If an additional track is needed in the layout action then at least one active net uses at most one track after the action is performed; hence at most $2d_{\max}-1$ tracks are used.)
fi
fi

Again the cost of the algorithm is clearly $O(1)$ per column and hence $\theta(n)$ altogether.

We gave simplified algorithms and proofs for three results on channel routing. The simplification also reduced the running time from $O(d_{\max}n)$ to $\theta(n)$ in one case.

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