2.16 Other Inference Systems

Instantiation-based methods for FOL:

- Partial instantiation;
- Resolution-based instance generation;
- Disconnection calculus.

Further (mainly propositional) proof systems:

- Hilbert calculus;
- Sequent calculus;
- Natural deduction.

Instantiation-Based Methods for FOL

Idea:

Overlaps of complementary literals produce instantiations (as in resolution);

However, contrary to resolution, clauses are not recombined.

Instead: treat remaining variables as constant and use efficient propositional proof methods, such as DPLL.

There are both saturation-based variants, such as partial instantiation [Hooker et al.] or resolution-based instance generation (Inst-Gen) [Ganzinger and Korovin],
and tableau-style variants, such as the disconnection calculus [Billon; Letz and Stenz].

Hilbert Calculus

Hilbert calculus:

Direct proof method (proves a theorem from axioms, rather than refuting its negation)

Axiom schemes, e. g.,

$$F \rightarrow (G \rightarrow F)$$

$$(F \rightarrow (G \rightarrow H)) \rightarrow ((F \rightarrow G) \rightarrow (F \rightarrow H))$$

plus Modus ponens:

$$\frac{F \qquad F \rightarrow G}{G}$$

Unsuitable for both humans and machines.

Natural Deduction

Natural deduction (Prawitz):

Models the concept of proofs from assumptions as humans do it (cf. Fitting or Huth/Ryan).

Sequent Calculus

Sequent calculus (Gentzen):

Assumptions internalized into the data structure of sequents

$$F_1, \ldots, F_m \to G_1, \ldots, G_k$$

meaning

$$F_1 \wedge \cdots \wedge F_m \to G_1 \vee \cdots \vee G_k$$

A kind of mixture between natural deduction and semantic tableaux.

Perfect symmetry between the handling of assumptions and their consequences.

Can be used both backwards and forwards.